SOCIETY OF PHYSICS STUDENTS
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# SPS Chapter Research Award Proposal

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| **Project Proposal Title** | Abnormal Shadow Distributions from Relativistic Light Sources |
| **Name of School** | University of Central Florida |
| **SPS Chapter Number** | 1076 |
| **Total Amount Requested** | $2000.00 |

## Abstract

SPS at UCF plans to find new, interesting results by applying Special Relativity to a problem traditionally solved in a simple Newtonian context. Doing so will enhance members' understanding of Special Relativity and produce novel results without requiring full expertise in advanced mathematics and upper-division physics.

# Proposal Statement

## Overview of Proposed Project

Mathematical distributions are important to physicists and appear in all types of problems. The ability to use distributions and understand them when they appear is key to solving complex problems. The most famous distribution that has been created, the Dirac delta function, has allowed for application to problems where it is deemed suitable. Being able to use distributions, like the Dirac delta function, is an easier problem than constructing them from scratch. Thus, a deeper understanding of mathematics will be needed to investigate new distributions that appear when solving physics problems.

In this research project, we propose to provide undergraduate students at UCF with an opportunity to understand mathematical distributions that arise in physical contexts, particularly towards Newtonian problems that use Special Relativity as their theoretical framework. Specifically, there will be two groups of students that will construct Newtowian problems and solve them with Special Relativity, with the intent that the result be unexpected. These problems will include mathematical distributions and require students to know the implications of constructed distributions in a physical context.

The proposed research project will strengthen the objectives of the SPS program by providing students with the opportunity to build skills needed to meaningfully contribute to the professional physics community. The collaborative nature of this project encourages effective interactions and communication within a group. In doing so, involved students will also form a strong network of peers with varied experience, whom they otherwise may not have met. Of course, participants will gain research experience--and through the dissemination process, students will have the opportunity to present their work at a conference or through a written report.

## Background for Proposed Project

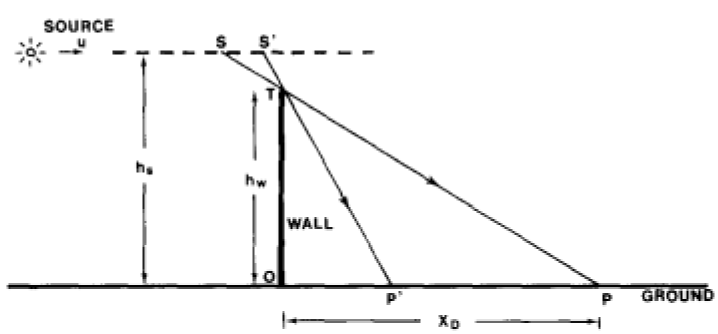
Finding unexpected results by solving a Newtowian problem with Special Relativity is not an easy task. Many problems could be constructed but not yield any interesting phenomena. Successfully doing this requires the use of mathematical distributions, which is a familiar idea in physics. There are many books dedicated to this theory, see [1], [2], and [4]. One paper that illustrates the ability to discover an unexpected result that includes a distribution, involves a moving light source and an object that casts a shadow, see [3]. The results of this paper demonstrate, by considering the finiteness of the speed of light, that a shadow (the distribution to be investigated) will not disappear continuously. Furthermore, the shadow will disappear earlier at a location closer to the object casting the shadow for specific criteria in Fig. 1, see [3].

The brilliance of this paper was the construction of the used problem because it allowed for one to solve it without increasing the mathematical complexity that could come when paired with Special Relativity. They were able to find the time it takes for a shadow to disappear, or the illumination of a part of the ground by a light source. Explicitly, this time function was found using the construction from [3],

$$t_D = \; - x_D(h_S - h_W)/(h_W u) + c^{-1}\sqrt{h_S^{\,2} + (h_S^{\,2} x_D^{\,2}/h_W^{\,2})}\,.$$

This equation was investigated to show that if there exists a minimum, then the shadow will have a hole in the distribution as it moves along the ground. This strange and unexpected phenomenon existed for only cases that satisfy the inequality

$$u/c \; > \; 1 - h_W/h_S\,.$$



There are two things one can take away from this paper: the importance of the construction of a problem, in that it will affect the possibility of solving and finding an unexpected result, and the question of what distribution can be created from the construction such that it can be investigated and analyzed for an unexpected result. One thing this paper did not investigate was the shadow's distribution of motion.

What we propose is to have students investigate the dynamics of this shadow's distribution when the unexpected result will exist, as one big group. We will then be able to create a function that describes this distribution and explain why it is not continuous. Later, once students have become accustomed to this problem, we will investigate newly constructed problems or old problems that can be approached with Special Relativity to investigate distributions and their consequences.

Overall, the field of Special Relativity encompasses this idea of finding unexpected results to problems that are deemed 'not interesting'. The proposed work will add to the field of Special Relativity since the goal of the project is to create and understand unexpected phenomena that appear when solving problems. Also, this project will add the field of mathematical distributions since we will be studying and understanding newly constructed distributions that arise in the problems we will be solving.

## Expected Results

We plan to produce a collection of results that are new and unexpected from simple Newtonian problems. For clarity, an unexpected result is defined as the following: a result of a physical problem that is counterintuitive based on the construction and real world bias of not experiencing life relativistically. So, the results would demonstrate the existence of phenomena that are hidden from us as humans since we will never understand things that are inherently relativistic. The results should also depend on mathematical distributions that come from the problem created. Thus, we will also collect old and new distributions that will appear in physical problems that can be used later in applications.

These results will be created in LaTeX intended for publications in journals and submitted for conferences. Thus, students involved will be exposed to and learn vital skills for their future academic career and allow for an opportunity in undergraduate research. All individuals active in this project will also build a strong foundation in Special Relativity and relevant Mathematics that can be used outside this project in future research and coursework. Also, these results allow for an important outreach opportunity since it provides an audience with simple problems, that do not require a strong background in physics, that come paired with results that highlight the significance of relativistic effects.

## Description of Proposed Research - Methods, Design, and Procedures

We plan to begin by reviewing the relevant aspects of Special Relativity, these would include Einstein's postulates and the consequences of Special Relativity of common Newtowian problems. Then we will review common mathematical distributions used in physics. Both of these types of review sessions will be held at designated meeting areas located on campus. These review sessions will occur, at a minimum, once a week and will start happening once we get funding to buy the relevant textbooks and restricted research papers so we can use these resources to teach fellow undergraduates in the important fields necessary to work for this project.

We then will go over a research paper that we will attempt to mimic in the future, see [3]. We will solve and work through the problem as a group and understand the process used in this paper: constructing a Newtowian problem, using Special Relativity, detailing a distribution, then achieving an unexpected result. We will meet once a week until this task has been completed.

After going through the paper as a group, we will make a contribution based on this paper. The author of [3] has not explicitly characterized the distribution of the shadow, which we can do in addition to explaining why a shadow disappears closer to the object casting a shadow. This will make sure everyone is on the same footing of knowing what is required to reach an interesting result by creating a problem from scratch or extending and modifying an existing problem. At this point in time we will have funding to access Wolfram Alpha and create graphics of the shadow's distribution. This will be reviewed by our faculty advisor before we finalize the result to be created into a research paper. Will we then write up this result in LaTeX, for it to later be submitted to journals and conferences. The step of writing up the results in LaTeX will be split into individuals so those not experienced in LaTeX can learn and receive help for academic writing in this editor. We will meet for this task once a week just like the previous task until it has been completed.

After the previous part has been accomplished as one large group, we will have all funding needed to do the rest of the project and split into two main groups. Each group will meet, on their own time, and create potential problems to be solved that include a distribution that could potentially create an unexpected result. Once each group has created a list of problems, we will meet as one big group and solve them together at a problem-solving party, which will include refreshments and food, since this will continue for hours. We will repeat this process twice, of creating problems and hosting a solving party so we end up with at least two problems that give novel results. We will pick at least one main problem to be written up in LaTeX for each group. These will be reviewed by our faculty advisor like the previous step and will be transformed into paper format to be submitted to journals and conferences in the future. Meeting times and structures for this task will vary because everyone involved will be well acquainted with this project's outcome. Thus, meeting for this final task will depend on the preferences of individual groups which will affect when the problem-solving parties occur.

## Plan for Carrying Out Proposed Project

We plan to form a cohort of entirely SPS members, but welcome outside participation. Based on current attendance and expressed interest, we anticipate a group of six members in addition to the two project leads. Some turnover is expected as contributors graduate. First, as one big group, we will go through the paper that inspired this whole project and rework all the results and conclusions that were made. Next, we will extend new work from this paper on the shadow's distribution that has not been investigated yet. Then, after each personnel has a strong understanding of what is possible and expected from investigating a Newtowian problem with Special Relativity, we will split into two groups and assign one problem per group to be investigated.

Olivia Bitcon will act as the project lead and primary point of contact, and Michael Lynch will serve as a co-lead. These project leaders together possess extensive research experience and sufficient Special Relativity background. Michael has taken Special Relativity as a course and is active in current research in this field, creating two papers to be published in journals and going to conferences to display such research. Also, Michael is taking graduate Electrodynamics, which requires one to apply Special Relativity to Electricity and Magnetism. Olivia has contributed to various research projects at the Florida Space Institute with a small group of students from different disciplines, which will prove useful since contributing students will be in different stages in their education. She has also conducted research in Neutrino Astrophysics and is writing an undergraduate thesis in Cosmology, providing a different perspective. Olivia has presented at many conferences, and contributed to our previous chapter research project while maintaining close contact with its leaders, David Wright and Spencer Tamagni. We have not required a firm commitment at this point, so there may be additional expertise coming from our future collaborators.

One option for research space is to reserve the Physical Sciences conference room, which has ample dry erase board space. This reservation process is familiar and done for chapter meetings. The undergraduate study room is also an option for smaller groups. Our project doesn't require any long-term physical space, and students could even work independently on pencil and paper from their own homes.

The department allows us access to the aforementioned conference room and undergraduate study room. One of our faculty advisors, Prof. Efthimiou, is a mathematical physicist that is well versed in the research field we intended to contribute to. We will be using his course material of Special Relativity, as well as his own written textbook, as a guide and inspiration for creating problems to be solved. Prof. Efthimiou has provided us with this project's theme and the research paper that most of the produced results will be based on. He will look over this project to keep this work credible and will exist as an excellent source for references for papers and textbooks that will help this project.

## Project Timeline

**January 2023**: Receive funds. Finalize item list and order budget items. Begin reviewing background to relevant fields of physics and mathematics with members.
**February 2023**: Continue reviewing Special Relativity background with members. Make textbooks accessible. Rework all calculations and conclusions from [1] and add a contribution based off of this paper as one team. Create a preliminary list of potential problems.
**March 2023**: Select a problem from the preliminary list and continue reviewing relevant background. Hold the first group problem-solving party.
**April 2023**: Organize and hold smaller problem solving sessions. Encourage independent contributions over summer.
**May 2023**: Prepare and submit interim report. May includes finals and the first few weeks of summer break
**Summer 2023**: Receive remaining funds and distribute remaining resources (textbooks/subscriptions) as possible.
**September 2023**: Resume activities, allow for involvement of new members. Second problem-solving party.
**October 2023**: Participants' solution is reviewed by advanced undergraduates and graduate volunteers. Decide on method of dissemination. Some options include submitting to UCF's undergraduate research journal, SPS's Journal of Undergraduate Reports in Physics (JURP), and UCF's annual Student Scholar Symposium.
**November 2023**: Solutions are further reviewed by faculty and compiled for distribution.
**December 2023**: Begin dissemination of results. Prepare and submit the final report.

## Budget Justification

Books will be made available to members and used by project leaders to explain necessary background. They will be especially useful since our modern physics courses do not cover Special Relativity in much detail. They will provide inspiration for creating problems, then used to help solve said created problems. We anticipate purchasing 4-8 books that cover the physics and mathematics needed for this project: Optics, Special Relativity, Newtowian physics, Mathematical Distributions, and Mathematical Analysis. Textbooks will vary in rigor but will focus on the fields stated above. For clarity, generalized functions and distributions are the same thing but are named differently by two different fields of study. Such books would be: Elementary Approach to Special Relativity by Günther, The Geometry of Special Relativity by Dray, Mathematical Methods in Physics: Distributions, Hilbert Space Operators, Variational Methods, and Applications in Quantum Physics by Blanchard and Brüning, Methods of the Theory of Generalized Functions by Vladimirov, Generalized Functions and Partial Differential Equations by Friedman, and two other books on Optics and the mathematical theory of distributions.

Some calculations are difficult if not impossible to solve analytically. We also need software to graph and plot high quality figures. We will address these needs by providing 12-month Wolfram Alpha subscriptions to five dependable, consistently involved students.

Food and beverages will be necessary to sustain students for the rather long problem-solving parties. In the past, this has kept up morale and interest as the semester becomes more difficult. We estimate $200 will cover 2 large sessions and 1 or 2 small sessions.

We do not anticipate funding or supplies from other sources.

# Bibliography


[1] Demidov, A. S. (2001). *Generalized functions in mathematical physics: main ideas and concepts* (Vol. 237). Nova Publishers.

[2] Gel'fand, I. M., & Shilov, G. E. (2016). Generalized Functions, Volume 1.

[3] Lai, H. M. (1975). Extraordinary shadow disappearance due to a fast-moving light source. *American Journal of Physics*, *43*(9), 818-820.

[4] Vladimirov, V. S. (1976). Generalized functions in mathematical physics. *Moscow Izdatel Nauka*.