Dear Editor,

Your article on amortization schedules was quite interesting. Your solution to the problem of how to understand mortgages is easily understood by anyone whose calculus abilities are not completely gone. As you point out, the result is a model of how mortgages work with some limitations. The primary problem, I think, is that a “continuous” payment is assumed in the integration of the differential equations to get a solution.

Your approach was to get an equation for the interest paid so that you could then compute the monthly payments. As you discovered, there is some difference between the n computed from your continuum model and the actual n derived from the discrete payments actually made in the real world. The real world n is easily derived without knowing the total interest paid.

Let M_0 be the loan amount, with M_j the loan amount left after payment is made in month j, n is the monthly payment, and I is the monthly interest rate, i = k/12, where k is the yearly interest rate.

So the remaining loan amount after the first month is

\[ M_1 = M_0 (1 + i) - n \]

The second and third months follow easily and the Nth month is seen to be

\[ M_N = M_0 (1 + i)^N - n \sum_{j=0}^{N-1} (1 + i)^j \]

The summation is a finite geometric series:

\[ M_N = M_0 (1 + i)^N - n \frac{1 - (1 + i)^N}{1 - (1 + i)} \]

Since the final payment N reduces the loan amount to zero, one can solve this equation for n:

\[ n = \frac{i M_0}{1 - \frac{1}{(1 + i)^N}} \]

One can see that the monthly payment is the monthly interest on the loan amount inflated by an amount depending on the monthly interest rate and the number of payments. Thus the monthly payment is just enough to start with so that after N payments the loan amount is reduced to zero.

It should be noted that n will be a real number with more than two significant digits after the decimal point. In other words, the payment will be rounded up to the nearest penny, since payments are not made in fractions of pennies. One other point that should be made is that the monthly payments and interest computations actually compound the interest rate. So what this means is that a 14.25% annual rate really becomes a 15.22% rate if one simply divides the annual rate by 12 to get the monthly rate. This seems to be what is done.

Sincerely,
Fred J. Kopp