

History of Big Bang Cosmology, Part 5: *Initial Conditions, Inflation, and Fluctuations*

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As you know, four main bodies of evidence affirm the big bang: the dark sky, Hubble expansion, element abundances, and cosmic microwave background radiation (CMBR). Parts 1-4 of this series briefly examined these topics. [1-4] Here we say a few words about the window the CMBR offers into the *very early* universe, including a brief but crucial epoch of “inflation.”

Recall that the CMBR was first predicted in 1948 by Ralph Alpher and Robert Herman.[5] They calculated its present temperature to be about 5 K, putting the peak of its Planck spectrum (“blackbody” radiation) in the microwave band. For nucleosynthesis calculations to achieve ~0.1% agreement with observations, it was necessary to assume a billion-to-one ratio of photons to baryons. These photons need not be postulated *ad hoc*. When the thermal energy per photon fell below mc^2 for some particle species, after pair annihilations a matter residue was left over along with an enormous photon population, due to prior particle/antiparticle decay rate asymmetries.

The radiation and matter maintained thermal equilibrium thanks to Thomson scattering between photons and charged particles. Helium synthesis occurred about 3 minutes after the big bang, and for the next 380,000 years the photon mean free path between successive Thomson scatterings remained very short. About 380,000 years after the big bang, the temperature fell to about 3000 K, cool enough to quench photoionization and allow electrons and nuclei to bind into neutral atoms. After this decoupling process, photons could propagate freely; the universe became transparent to radiation. This radiation preserved its Planck distribution, with ever-decreasing

temperature, as the universe continued to expand. (Recall the time since the big bang and the temperature were related approximately by $T^2 t \approx 2 \times 10^{20} \text{ K}^2 \text{ s}$ in the early universe.)

Throughout the 1950s, Alpher and Herman tried to convince radio astronomers to search for this CMBR. In retrospect, it evidently *had* been observed by A. McKellar in 1940-41 but not recognized;[6] it was observed and recognized by Tigran Shmaonov who reported $T = 4 \pm 3 \text{ K}$ in his 1957 doctoral thesis and a Soviet journal, but this report was not widely noticed.[7] Not until the mid-1960s was the CMBR observed *and* correctly interpreted *and* the news widely disseminated. Apparently unaware of prior predictions and measurements, in 1964 Arno Penzias and Robert Wilson of Bell Labs, while calibrating an antenna to receive microwaves bounced off the Echo satellite, found anomalous radio noise with a temperature of 2.7 K. Their finding was immediately understood by Robert Dicke and his colleagues at Princeton to be the CMBR. They had been building an antenna to search for this radiation. The two groups’ papers appeared together in 1965.[8,9] The pace of CMBR research was thrown into high gear.

In the 1970s NASA invited proposals for astronomical research instruments to be carried aboard satellites. To cut a long story short,[10,11] John Mather of the Goddard Space Flight Center and George Smoot of UC-Berkeley became principal investigators in the design, construction, and use of the Cosmic Background Explorer (COBE) satellite, built

to make precision measurements of the CMBR. Over a thousand people worked on COBE. It carried three main instruments. The two that concern us here are the Far Infrared Absolute Spectrophotometer (FIRAS) and the Differential Microwave Radiometer (DMR). FIRAS was built to show the smoothness in the CMBR, and the DMR to show temperature fluctuations farther out in the decimal places.

Mather led the FIRAS experiment. It measured the CMBR spectrum in wavelengths from 0.5 mm to 1 cm, which includes essentially all the active CMBR spectrum. This measurement was essential to nail down conclusively the big bang nativity of the CMBR. Before FIRAS, only fragments of the CMBR spectrum had been fit to a Planck curve. Smoot led the DMR experiment. That instrument’s design featured dual receiver horns to measure the temperature *difference* between two points of sky. Such radiometers were pioneered by Robert Dicke in the 1940s.

COBE was launched in November 1989, with Ralph Alpher and Robert Herman in attendance. Within a few weeks the FIRAS team had collected enough data to announce a “preliminary measurement” of the CMBR spectrum.

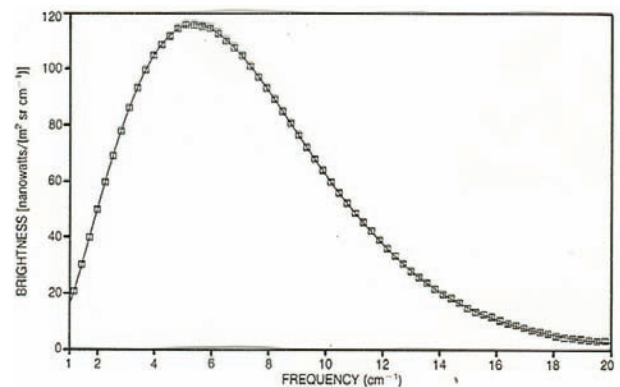


Fig. 1. The CMBR spectrum of the 1990 COBE-FIRAS results, fit to a Planck distribution at a temperature of $2.735 \pm 0.06 \text{ K}$. From Mather et. al., Ref. [12]. Reproduced permission of the American Astronomical Society.

Mather made the announcement at the January 1990 meeting of the American Astronomical Society. Subtracting out our galaxy's background and the dipole from our local system's peculiar motion, Mather reported $T = 2.735 \pm 0.06$ K,[12] and presented the graph shown in Fig. 1, which the audience greeted with a standing ovation.

The error boxes in Fig. 1 are astonishingly small, and the fitted Planck curve passes through *every single one* of them. To its level of precision FIRAS showed the CMBR temperature to be uniform across the sky. Additional FIRAS data published in 1994 increased the precision to $T = 2.726 \pm 0.010$ K.[13]

This uniformity affirmed the "unperturbed" big bang model of Friedman, Lemaître, Robertson, and Walker, which assumes isotropy and homogeneity at the largest scale. Within the framework of General Relativity this assumption allows only a time-dependent rescaling of space. In spherical coordinates, and allowing for non-Euclidean geometry (here suppressing latitude and longitude), the invariant proper time interval $d\tau$ between events gets parameterized as

$$c^2 d\tau^2 = c^2 dt^2 - R^2 (1 - k\chi^2)^{-1} d\chi^2 \quad (1)$$

where t is the time since the big bang as measured by the wristwatch of any "comoving" observer (defined below) and c = speed of light. If space is "flat" (Euclidean) then $k = 0$; if spherical (hyperbolic) then $k = +1(-1)$. $R = R(t)$ is the length-dimensioned "cosmic scale factor." χ is a dimensionless coordinate-grid label that identifies a particular circle centered on the origin. The *distance* between two circles is $\int R(1 - k\chi^2)^{-1/2} d\chi$. In the expansion R increases with time, swelling the distance between two grid-markers of fixed χ . $R(t_2)/R(t_1)$ gives the factor by which the universe expanded between two times.

Eq. (1), put into Einstein's field equations with cosmological constant Λ , yields the Friedmann equation for the evolution of $R(t)$:

$$H^2 + k/R^2 = \kappa\rho + \Lambda/3 \quad (2)$$

where $\kappa \equiv 8\pi G/3c^2$, G = Newton's gravitation constant, and ρ = energy density. Hubble's parameter is defined as

$$H \equiv \dot{R}/R \quad (3)$$

where $\dot{}$ denotes time derivative. Recall the universe expands according to "Hubble's law." At large scales the relative velocity v between a pair of galaxies is proportional to the distance D between them, $v = HD$, and $H_{\text{now}} = 70.1 \pm 1.3$ (km/s)/Mpc, although it varies across cosmic history.[14] The velocities in Hubble's law are *not* like *walking across* a rubber sheet, but instead are analogous to *being carried on a stretching* rubber sheet. Such "carried-along" motion is called *comoving*. Fall into orbit about your galactic center and you acquire a "peculiar" velocity.

If H were constant across cosmic history, then the time elapsed since any pair of points in the universe separated (since the big bang itself) would be $\Delta t = v/D = 1/H_{\text{now}} \approx 14$ Gyr. At any epoch, $1/H$ defines the *Hubble time*, and c/H the *Hubble distance* that light travels during this time, offering a size scale for the observable universe. Placing ourselves at the origin, the χ *coordinate* on the other side of the Hubble distance is a quantity we will need later,

$$\chi_{\text{Hubble}} = c/RH = c/\dot{R}. \quad (4)$$

Einstein's field equations along with energy conservation give a second relation for R ,

$$\ddot{R} = -\frac{1}{2} \kappa(\rho + 3P)R \quad (5)$$

where P denotes the pressure. The energy density and partial pressure of constituent i are related by an equation of state,

$$P_i = w_i \rho_i \quad (6)$$

where i denotes matter, radiation, or

anything else we might encounter. For nonrelativistic matter $w_m = 0$, and $w_r = 1/3$ for radiation. We absorb Λ into a density by introducing ρ_Λ where X denotes Λ or some constituent of the universe that offers an *effective* cosmological constant (of course, both could exist). If $X = \Lambda$ then $\rho_\Lambda = \Lambda/3\kappa$ and $w_\Lambda = -1$. Λ is too small to dominate physics in the *early* universe; this we know from its negligible influence in General Relativity effects such as perihelion precession.

To see which constituents dominate across time, write the first law of thermodynamics for an adiabatic expansion of the universe, noting that volume $\sim R^3$:

$$d(\rho R^3) = -PdR^3. \quad (7)$$

Insert the equation of state to find $\rho_i \sim R^{-3(1+w_i)}$, which in Eq. (2) (neglecting k) leads to $R \sim t^{1/2}$ for radiation, $R \sim t^{2/3}$ for matter, and $R \sim e^{\lambda t}$ for Λ where $\lambda \equiv \sqrt{\Lambda/3}$. Because $T \sim 1/R$, radiation dominates in the very early universe; matter dominates after photon decoupling, and Λ (if > 0) dominates in an old universe. The corresponding Hubble coordinate velocities evolve as $\dot{\chi}_{\text{Hubble}} \sim t^{-1/2}$ for a radiation-dominated era; $\dot{\chi}_{\text{Hubble}} \sim t^{-2/3}$ for matter; $\dot{\chi}_{\text{Hubble}} \sim -e^{-\lambda t}$ for Λ . In a race between light and coordinate grid lines "painted on the rubber sheet," the grid lines pull ahead. No violation of Relativity occurs because no signal "walks" across the "rubber sheet" faster than light.

To compare the model to reality one must express these relations in terms of observables. A "flat" universe has $k = 0$, and with $\Lambda = 0$ Eq. (2) defines a "critical energy density" $\rho_{\text{cr}} \equiv H^2/\kappa$. Define $\Omega \equiv \rho/\rho_{\text{cr}}$, the ratio of actual to critical energy density, and do likewise for each constituent species, $\Omega_i = \rho_i/\rho_{\text{cr}}$. Although it's not really an energy density, define also the "curvature density" $\Omega_k \equiv k/R^2H$. Now Eq. (2) may be elegantly written

$$1 = \Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k \equiv \Omega + \Omega_k. \quad (8)$$

Notice that $\Omega = 1$ in a flat universe.

If only matter and radiation filled the universe, then the expansion would continuously decelerate. With the “deceleration parameter” $q \equiv -\ddot{R}/RH^2$, Eq. (5) may be neatly written

$$q = \frac{1}{2} [\Omega_m + 2\Omega_r + (1 - 3w_x)\Omega_x]. \quad (9)$$

The densities in Eqs. (8) and (9) can be written in terms of their present values and as a function of redshift. The fractional change in wavelength between source and receiver *defines* the “redshift,” $z \equiv (\lambda_{\text{rec}} - \lambda_{\text{emit}})/\lambda_{\text{emit}}$. The stretching of space itself follows the same pattern, $R_{\text{now}}/R_{\text{then}} = 1+z$. From thermodynamics, the expansion is adiabatic and $R \sim 1/T$. At the time of decoupling the CMBR temperature was 3000K, and we receive it today at about 3K, which puts the surface of last Thomson scattering at $z \sim 1000$. For instance, Ω_i at any redshift can be written $\Omega_i(z) = \Omega_{i,\text{now}}(H_{\text{now}}/H(z))^2(1+z)^4$. Thereby can the present values of H , q , Ω , and the Ω_i be related to specific models of the early universe’s composition.

The parts-per-thousand uniformity measured in the CMBR was a crucial affirmation of the big bang model. But this success, like all successes, allowed other interesting problems to surface, such as the kinematic “horizon” and “flatness” problems, and the dynamical problem of structure formation.

The Horizon, Flatness, and Structure Problems

With ourselves located at the origin $\chi = 0$, consider the propagation of a ray of light, emitted by a comoving source at time T . Let the light ray travel radially to us where we receive it at time t (the big bang event defines $t = 0$.) Because $d\tau = 0$ for light, the χ -coordinate of the *emission* event follows from Eq. (1):

$$\chi_{\text{em}}(t, T) = \int_T^t dt'/R(t'). \quad (10)$$

If χ_{em} diverges as $T \rightarrow 0$ that’s good because that means we can in principle

receive signals emitted from events arbitrarily close to the big bang itself; our “view” of the past universe has no limit. But if χ_{em} is finite as $T \rightarrow 0$ then $\chi_{\text{em}}(t, T)$ can never exceed the finite value $\chi_{\text{em}}(t, 0)$ for time t . [15] Then our view of the past universe would be limited by this so-called “particle horizon.”

The problem manifests itself when we consider that other observers elsewhere in the universe, who place themselves at the origin of *their* coordinate systems, would find the same result. We and they would be able to see *some* of the other person’s sky, but not *all* of it, yet we both measure the *same* CMBR temperature for our *entire* skies. How can two skies have the same temperature but be causally disconnected? Such a uniform CMBR temperature should emerge as a *consequence* of the model, and not have to be *postulated* as an initial condition. This is the “horizon problem.” Notice from Eq. (10) that the horizon problem arises if $R \sim t^n$ where $n < 1$, i.e., $\ddot{R} < 0$ unrelentingly through cosmic history.

A related problem asks: Why is the universe so flat? The observed value of the total Ω is very nearly (if not exactly) equal to 1. Why should this be puzzling? Because if $\Omega = 1 + \epsilon$, then $|\epsilon|$ grows *rapidly* with time. For $|\epsilon|$ to be close to zero today, when $t \sim 10^{18}$ s, would also require fine-tuning the initial conditions.

But if an epoch existed in cosmic history when $\ddot{R} > 0$, a loophole through these problems lies open. For instance, if R had a sudden burst of enormous growth, then the k/R^2 term in Eq. (2) would quickly become indistinguishable from zero, the universe *driven* to flatness independent of initial conditions. An epoch of positive acceleration is called a time of “inflation.” Eqs. (4) and (5) imply that $\dot{\chi}_{\text{Hubble}} < 0$ and $P < \rho/3$ during inflation.

Only one tiny problem remains: to find a mechanism within nature that makes $\ddot{R} > 0$. Clearly a small positive Λ would give an accelerating universe later in cosmic history, when the energy densities of radiation and matter have

thinned out, leaving ρ_Λ to dominate the energy budget. But that would not solve the horizon problem in the *early* universe. Not only is Λ too small, but it also *repels* (negative pressure!) so matter won’t clump to it—not an effective path to galaxy formation!

Although we can model the unperturbed universe with a smoothed-out energy density, the *real* universe shows a hierarchy of structures. Galaxies, clusters of galaxies, and superclusters separated by vast voids exist. How can such a smooth CMBR be reconciled with lumpy matter?

In a 1967 paper by R.K. Sachs and A.M. Wolfe we overhear the conversation already underway on this issue: “*The actual universe is quite lumpy, but the usual cosmological models assume a uniform distribution of matter. One simple method for making somewhat more realistic cosmological models is to consider linear perturbations away from spatially homogeneous isotropic models.*” [16]

Given fluctuations in an otherwise smooth primordial density, $\rho = \rho_0(1 + \delta)$ where $|\delta| \ll 1$ (sub- ρ_0 denotes “unperturbed background”), gravity would attract matter into regions where $\delta > 0$, draining matter from regions where $\delta < 0$, offering positive feedback to the fluctuations.

Before photon decoupling, what could be efficiently attracted into the overdense regions while baryonic matter was continually kicked around by Thomson-scattering with photons? Here the necessity of “cold dark matter” (CDM) seems compelling even though nobody yet knows the identity of the CDM particles themselves! They must be uncharged so they are not dispersed by all those photons. They must be “cold” (nonrelativistic) and heavy, so all forms of matter could be attracted to them as early as possible. The CDM must be non-baryonic, because the nucleosynthesis calculations place tight restrictions on baryon abundances, and they’re accounted for. [17]

The inhomogeneities, once formed,

must leave their imprint on the CMBR. Before photon decoupling, as matter collects in an overdense region the Thomson-scattering photons get squeezed. Their pressure and temperature shoot above ambient conditions. The overpressure pushes back on the surroundings like a spring, opposing the compression. With a period determined by the size of the region being compressed, these compressions and rebounds drive oscillations—sound waves forming a harmonic series of standing wave modes—until decoupling occurs. As adiabatic compressions and rarefactions, these perturbations must show up in the CMBR temperature field.

In the early universe, the speed of adiabatic sound waves in the plasma-photon medium is $c/\sqrt{3}$. Photon decoupling at $t_D \approx 380,000$ yr sets the scale for the fundamental mode: $L_D = t_D c / \sqrt{3} \approx 2 \times 10^5$ light-years, half the wavelength of the fundamental acoustical mode at that time. Today, the CMBR radiation comes to us redshifted to $z \approx 1000$. Therefore the fundamental mode region will have stretched since decoupling to a size $L_D z \approx 2 \times 10^8$ light-years. The photons reaching us today from the last Thomson-scatterings have been en route about 14 Gyr. A region originally of scale L_D will now appear to us (assuming Euclidean geometry) to subtend an angle on the sky $\Delta\theta = (2 \times 10^8 c \text{ yr}) / (14 \text{ Gc yr}) \approx 0.014 \text{ rad} \approx 1^\circ$. The overtone wavelengths are $1/2, 1/3, 1/4, \dots$ of the fundamental, and should peak at $1/2^\circ, 1/3^\circ, 1/4^\circ, \dots$. For example, the third harmonic corresponds to a region of size $L_D/3$ at decoupling, where there was just enough time for one squeeze followed by one rarefaction and a second squeeze. Temperature fluctuations in the sound waves that existed then should reveal themselves now as peaks and valleys in a graph that plots the CMBR power spectrum (averages of the harmonic amplitudes squared) against angular separation. (These fluctuations are mapped on the celestial sphere as a multipole

expansion in “spherical harmonics.”)

When the possibilities of density fluctuations seeding cosmic structure were first studied, finding a mechanism to produce the original density fluctuations, without having to postulate them as initial conditions, was a real question. James Peebles and J.T. Yu closed a 1970 paper worrying about this point: “*It is well to bear in mind that in this calculation the initial density fluctuations are invoked in an ad hoc manner because we do not have a believable theory of how they may have originated...*”[18]

Happily, quantum mechanics requires fluctuations—however microscopic—in any observable. If you accept quantum physics, fluctuations are not optional. Admittedly, it’s a bold leap to suggest that microscopic quantum fluctuations triggered structure formation across the entire universe! To understand the evolution of these fluctuations we must go back deep into the very early universe—to about $t \sim 10^{-35}$ s. The horizon and flatness problems, and the amplification of the quantum fluctuations, share a common solution.

Inflation

Suppose a spatially uniform scalar field $\Phi(t)$ permeates the entire universe.

[19] By comparing the relativistic field theories of Φ and an ideal fluid, the energy density and pressure carried by Φ are

$$\rho_\Phi = \frac{1}{2} \dot{\Phi}^2 + V(\Phi) \quad (11)$$

and

$$P_\Phi = \frac{1}{2} \dot{\Phi}^2 - V(\Phi) \quad (12)$$

where $V(\Phi)$ denotes a model-dependent potential. Most models use a potential that looks something like Fig. 2(a).

Should the scalar field energy density dominate over all other constituents, then Eqs. (2) and (5) become

$$H^2 = \kappa [\frac{1}{2} \dot{\Phi}^2 + V(\Phi)] \quad (13)$$

$$\ddot{R} = -\kappa R [\dot{\Phi}^2 - V(\Phi)]. \quad (14)$$

Eliminate \ddot{R} between Eqs. (13) and (14) to obtain

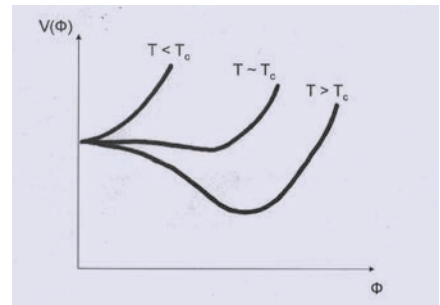
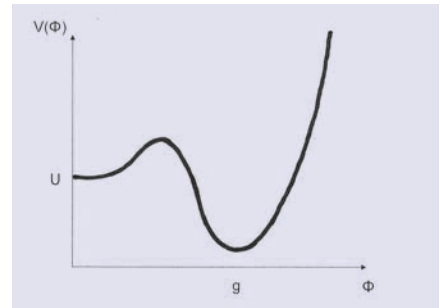


Fig. 2. (a) Schematic potential $V(\Phi)$ for the scalar field Φ . Note that when $\Phi = 0$ then $V(0) = U$, the “false vacuum,” and that $V = \min.$ when $\Phi = g$. (b) Temperature-dependent potential.

$$\ddot{\Phi} + 3H\dot{\Phi} = -dV/d\Phi \quad (15)$$

which resembles $F = ma$ for a mass sliding with gravity and friction down a curve: $-dV/dx - b\dot{x} = m\ddot{x}$.

If we want the scalar field to produce inflation, $\ddot{R} > 0$ requires $\dot{\Phi}^2 < V(\Phi)$. Let’s push this to an extreme. Suppose that $\dot{\Phi}^2 \ll V(\Phi)$, or equivalently $\ddot{\Phi} \ll \frac{1}{2}dV/d\Phi$. From the mechanical analogy this assumption bears the name “slow roll approximation” because the peak of $V(\Phi)$ would be very broad and almost flat. Then Eqs. (13) and (14) are simply

$$H^2 \approx \kappa V \quad (16)$$

and

$$3H\dot{\Phi} \approx -dV/d\Phi. \quad (17)$$

Recall $H = \dot{R}/R$, and during the slow roll (when $V \approx \text{const.} \approx U$) Eq. (16) integrates to $R(t) \sim \exp[t \sqrt{\kappa U}]$. This exponential inflation in R , and exponential decline in temperature, $T \sim 1/R$, continues until V drops into its deep well, and by Eq. (17) any oscillations about $\Phi = g$ damp out because of the “friction,” and Φ reaches its ultimate minimum $V_{\min} \geq 0$. Φ

sliding down into the well corresponds to the decay of the scalar field's energy into more familiar particles, releasing latent energy $\rho_\phi R^3 \sim UR^3$ which reheats the universe. When does this happen and how long does it take?

Inflation models typically propose temperature-dependent coefficients in the potential, such as

$$V(\Phi, T) = a(T)\Phi^2 + b(T)\Phi^4. \quad (18)$$

The parameters in a and b may be chosen such that a critical temperature T_c exists (see Fig. 2b). For $T > T_c$ the most stable state resides at $\Phi = 0$ with $V = U$. When $T \sim T_c$ the “slow roll” begins, and for $T < T_c$ the potential well becomes available, where the most stable state resides at $\Phi = g$ with $V = V_{\min}$. When $\Phi = g$ the Φ quantum's interactions with other particle species become masses for the latter. To give quarks and leptons their masses the “Grand Unified Theories”[20] need $T_c \sim 10^{27}$ K, which occurred around 10^{-35} s. Using these numbers as an illustration, and taking $U \sim T_c^4$ (in the very early universe, all particles are ultra-relativistic, so $\rho \sim T^4$ as in Stefan's law), inflation proceeds with doubling time $\sim (\ln 2)/\sqrt{(\kappa U)} \sim 10^{-34}$ s. Suppose (in an extreme case) that inflation continues until $T \sim 1$ K. From Eq. (2) with $R \sim 1/T$, the elapsed time is $\Delta t = (\ln 10^{27})/\sqrt{(\kappa U)} \sim 10^{-32}$ s, a hundred doubling times. So during this “supercooling” era the scale factor R increases by $\sim 2^{100} \approx 10^{30}$. All this has been completed when the universe has aged to about $t = 10^{-35}$ s + 10^{-32} s $\approx 10^{-32}$ s. At the end of the slow roll, the phase transition occurs, releasing latent energy in an orgy of particle-antiparticle production, reheating the universe to $\sim 10^{27}$ K. Then it expands “normally” as the Friedmann equation describes.

Any quantum fluctuations have been hugely inflated, by a factor of 10^{30} in our simplified estimate. A 10^{-15} m region inflates to 10^{15} m, from the size of a proton to 0.1 light-year! Let's see how the CMBR offers a window into these mind-blowing events.

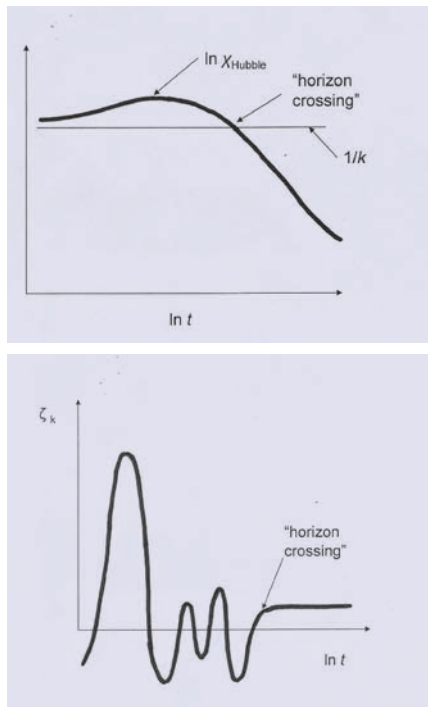


Fig. 3. (a) Evolution of χ_{Hubble} with time and (b) the transition of a mode from oscillating to constant when crossing the horizon.

Fluctuations

Introduce into the scalar field some quantum fluctuation, $\Phi = \Phi_0 (1 + \delta)$ with $|\delta| \ll 1$ and Φ_0 the unperturbed field. To first order in δ Eq. (16) says

$$H^2 \approx \kappa [V(\Phi_0) + \delta dV(\Phi_0)/d\Phi]. \quad (19)$$

Perturbations in the scalar field produce perturbations in its energy density and pressure, which are gravity sources in Einstein's field equations. Therefore the coefficients of dt and $d\chi$ in Eq. (1) acquire a “curvature perturbation” ζ . It is found in terms of t and χ by inserting the perturbed version of Eq. (1) into Einstein's equations. Then expand ζ in a harmonic series. Let ζ_k denote the amplitude of the harmonic with comoving wavenumber $|\mathbf{k}| \equiv k$ (not to be confused with the curvature index k which we don't need for now). Because wavenumbers are inverse lengths, the comoving and physical values are related by $k_{\text{physical}} = k/R$.

Place a node of a harmonic wave

at the origin. The other end of one comoving wavelength occurs at the coordinate $\chi_k = 2\pi/k$. When the wavelength of a mode happens to be much smaller than the Hubble distance, so that $\chi_k \ll \chi_{\text{Hubble}}$, this mode oscillates with time-dependent wavenumber k/R . On the other hand, when $\chi_k \gg \chi_{\text{Hubble}}$, the mode's wavelength is too large to fit within the Hubble distance, so the mode “turns off” and $\zeta_k \approx \text{const}$.

Meanwhile, during inflation χ_{Hubble} monotonically decreases (from Eq. (4), during inflation $\dot{\chi}_{\text{Hubble}} \sim -e^{-\lambda t}$ where $\lambda = \text{const}$. as the coordinate grids are superluminally stretched apart). During this time, fewer and fewer modes remain within the Hubble distance. Whenever $\chi_k = \chi_{\text{Hubble}}$, an event called “horizon crossing” (not the *particle* horizon) occurs: mode k makes a transition between oscillation and quiescence. Recalling the behaviors of χ_{Hubble} in different epochs, we see that a mode's wavelength may start out within the Hubble distance, but as the Hubble horizon diminishes that mode “freezes out,” as illustrated in Fig. 3b.

Later the Hubble horizon may grow to encompass the mode again, which resumes oscillating. Each mode that re-enters the Hubble horizon starts oscillating from zero “velocity.” Such coherence makes possible the peaks in the power spectrum of the temperature fluctuations.

So during inflation $R(t)$ increases exponentially while χ_{Hubble} decreases exponentially. Successive modes cease oscillating. Meanwhile, microscopic regions of space that experienced quantum fluctuations of the scalar field are inflated to macroscopic size, and cold dark matter falling into these regions carries structure formation forward. After inflation ceases at $t \sim 10^{-32}$ s, the universe enters the radiation-dominated era, where R and χ_{Hubble} both increase as $t^{1/2}$. Eventually χ_{Hubble} catches up to χ_k and the k th mode starts oscillating again. When photon decoupling occurs at $t \approx 380,000$ yr to initiate the matter-dominated era, the temperature variations from the adiabatic compressions and rarefactions

of the acoustical modes are preserved in the light from the last Thomson scatterings.

Finding evidence for these perturbations in the CMBR would take more decimal places, and thus more data, than the measurement of its 2.7 K temperature. By early 1993, sufficient data had been collected from the DMR instrument aboard COBE. At the April meeting of the American Physical Society that year, Smoot presented the DMR results to an expectant crowd. Many there hoped for a report on the presence of perhaps the quadrupole term. Smoot and his colleagues dramatically presented the multipole values one at a time, beginning with quadrupole—then 18 more multipoles beyond it![21]

Their efforts have since been beautifully extended by other groups working hard to fill in the CMBR power spectrum. These groups include the DASI interferometer stationed in Antarctica,[22] the balloon-borne Boomerang [23] and MAXIMA[24] instruments, and the Wilkerson Microwave Anisotropy Probe (WMAP) [25]. A recent WMAP update presented this power spectrum (Fig. 4):[26]

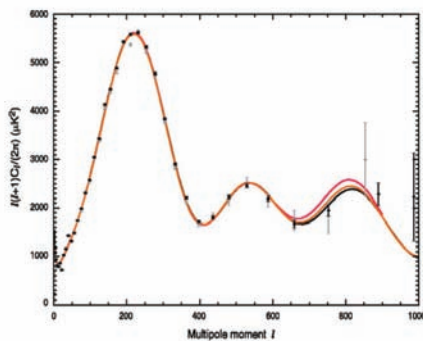


Fig 4. The CMBR power spectrum, Fig. 2 of Spergel et. al., Ref. [26]. Reproduced permission of the American Astronomical Society. The black line is best 3-year WMAP fit to a cosmological model with CDM and cosmological constant; the orange line combines 1-year WMAP data with data of other groups.

The height, width, and location of the various peaks and valleys of the power spectrum can be predicted for various models, and compared to measured values of parameters including H , q , Ω , Ω_b , curvature, Λ , and other observables.

Data from experiments such as DASI, Boomerang, MAXIMA, and WMAP have so far affirmed the “ Λ CDM” paradigm, an inflationary, flat universe with CDM and positive Λ (or its equivalent). For instance, an integrated WMAP survey reports $\Omega = 1.0052 \pm 0.0064$, $\Omega_\Lambda = 0.721 \pm 0.015$, $H = 70.1 \pm 1.3$ (km/s)/Mpc, $\Omega_{\text{baryon}} = 0.0462 \pm 0.0015$, age of universe = 13.73 ± 0.12 Gyr.[26,27] We have entered the era of *precision cosmology*.

With such precision in hand, the big bang stands as verified as any model can be for the foreseeable future. Now the problem gets turned around: the power spectrum has become a *precision tool* for studying the early universe. One of the most astonishing discoveries of all recently suggests the expansion is *accelerating*. We seem to be in another inflationary era, milder but longer-lasting than the spectacularly violent one at $t \sim 10^{-32}$ s. Consistent with the accelerating universe is the observation that Ω_Λ now dominates the energy budget, raising the astonishing notion that the energy density of empty space controls cosmic dynamics! Cosmic acceleration and the vacuum energy will form the last installment of this series.

Acknowledgments

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