Ralph Asher Alpher—Before the Big Bang

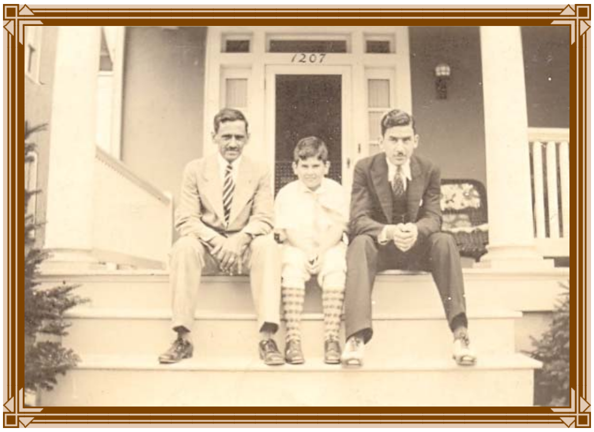
FEATURE

*by Victor S. Alpher, PhD**

Ralph A. Alpher was a co-founder of Big Bang cosmology, for pioneering calculations linking the expansion of the universe to primordial nucleosynthesis, and predicting the existence of the cosmic background radiation. We are honored to have his son, Victor S. Alpher, tell us his story. This is Part 1.

On February 3, 1921, a child was born—an unexpected son of Eastern European immigrants, Ralph Asher Alpher. His parents spoke Russian and Yiddish at home. One translation of his middle name Asher means "blessed." Were we blessed?

Ralph excelled in school—perhaps not unusual at the time for a son of immigrants. He always said that his older brother Robert—17 years older—was almost a father to him. Ralph graduated from Theodore Roosevelt High School in the District of Columbia at the age of 15. As the stage manager at the school, the money he brought in often supported his family. Ralph's mother passed away when he was only 17—and he was not told why at the time. It was sudden, and traumatic.



Samuel Alpher (father), Ralph Asher Alpher, and Robert J. Alpher (brother), on the steps of 1207 Farragut Street, the Alpher residence in NW Washington, DC, about 1930, when Ralph was 9.

(Photo provided to the collection of Ralph A. Alpher by Rita Alpher Glazer, printed here courtesy of Victor S. Alpher.)

During his school years, he discovered his attraction to the sciences, first to astronomy by one extraordinary school teacher—English teacher and amateur astronomer Matilde Eiker. Sarah Branch, a chemistry teacher, also stimulated his scientific mind, as did Richard Feldman, who introduced Ralph to physics. However, Ralph also became an excellent mathematician—a prodigy, apparently—quickly reaching the level at which mathematics is like a second language.

Gregg Shorthand: One Key to Success in the 1930s?

Around the time he finished high school, Ralph also completed a course in Gregg Shorthand. On his personnel papers he always stated that he could type 100 words per minute. He applied for a job at the Carnegie Institute in Washington, becoming secretary to the Director. Ralph also worked in 1940 as a statistical clerk and abstract clerk in the U.S. Bureau of Immigration and Naturalization and in the War Department. Because a paid job was vital for his family, he started college at the night school of George Washington University (GWU). As an adjunct to his secretarial work, he also had his first physics "assignment" assisting Scott E. Forbush in his research on cosmic rays (important today for survival of astronauts conducting space walks).

It was not long before the government began clandestinely to prepare for war. Much of the work was sequestered secretly in the Department of Terrestrial Magnetism (DTM) of the Carnegie Institute. DTM later became Section T of the National Defense Research Committee (NDRC) under the direction of Vannevar Bush.[1] Ralph was destined to work in this secret unit.

With his appointment as a physicist, Ralph came under the responsibility of distinguished physicist Merle Tuve from Yale University. Tuve had been brought down to DC in 1940 to oversee technical and experimental research under Vannevar Bush. In Section T were gathered prominent physicists and engineers from across America, as well as many early in their careers. Ralph began his career there working on an important project—the degaussing of ships.

America was overtly neutral at that time. Marine General Smedley (possibly the most widely popular General until George S. Patton's exploits) had recently published a small book, *War is a Racket*.[2] While pacifists marched in the streets, the government prepared for war. The first keel for a warship intended for the coming conflict had been laid in 1939. Involvement in any foreign conflict would be very unpopular—until December 7, 1941.

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As many men did, Ralph attempted to enlist in the military. He had been the commanding Major of the Cadet Corps at Theodore Roosevelt High School. His choice was the Navy, where as a college student he could be accepted into a shortened Officer Training School. Nonetheless, he was quite nearsighted, enough to deny him entrance into the military.

In the early 1940s Ralph Alpher could hardly imagine where his career as a physicist would ultimately lead. But only in his early 20s, he was already working on the most important scientific and technical projects in the world.

Ralph Asher Alpher in 1942, at the age of 21. By this time, he was already working on secret projects for the US Navy in areas such as degaussing (electromagnetic countermeasures to induced polarity) of seagoing vessels for the Navy and Merchant Marine.

(Photo courtesy of Victor S. Alpher).

The growing Section T had been relocated to 8621 Georgia Avenue in Washington, DC. This relocation gave space and secrecy to a group that would total 800 employees at its height, with over 15,000 technical reports produced during the War. Previously, 8621 Georgia Avenue had been an automobile dealership. But there were no new civilian automobiles produced from 1942 through 1945. Gasoline and many other items were rationed. Construction at the site was classified—any renovations of existing buildings or construction of new buildings had been curtailed for the duration of the War.

Merle Tuve ran an unusually intelligent workforce. Tuve's watchwords were: "I don't want you to save money, I want you to save time!!" The American workforce quickly became mobilized for war. By 1945, American workers would be turning out thousands of tanks, planes, and military vehicles per week. In 3 ½ years after entering the war, the United States decisively helped bring World War II to a close.

Section T was doing its work for the Navy through the Naval Ordnance Department and the Bureau of Ships. Ralph was spending a good deal of time at the Washington Navy Yard. As a junior physicist deeply involved in the degaussing of merchant and naval vessels, he was busy as he could be.

Ralph married Louise Ellen Simons, also a GWU student, in 1942, and finished his BS in 1943. In 1945, he finished his Master's Thesis, and in 1948 his Doctoral Thesis. During these years he spent many lunch hours at 8621 Georgia Avenue in Silver Spring, MD, with George Gamow, professor at GWU.

George Gamow agreed to take Ralph on as a graduate student. They began discussing Gamow's favorite subject, the creation of the universe. At that time, cosmology was not a popular topic in physics. With Gamow, Alpher became part of an academic genealogical line going back to the the University of Leningrad. There, Gamow had studied under the prominent Russian physicist Alexander Friedmann. Friedmann had been an original proponent of what became known as the Big Bang theory. But let us return to Ralph's work during the War.

Direct Threat to the Mainland

The first U-boat of Operation Drumbeat set sail from Germany on December 18, 1941, just over a week after the Japanese attacked Pearl Harbor. German U-boats were sinking ships laden with petroleum and gasoline right off the coast of the eastern United States by early 1942.[3] This was the first such direct threat to the mainland US since the War of 1812. There was no time to waste. Now entrances to US harbors were also being mined by Germany, which introduced magnetic mines during this war. They were initially sensitive to 15 gauss, and by 1943, to 4 gauss. Shipping to Great Britain was threatened by U-boats in the sea lanes of the North Atlantic, and much military equipment and foodstuffs were lost to the Atlantic deep.

Degaussing was effective and critical work. Ralph once explained the fundamental problem to me (of course, over coffee). Large steel ships are like bar magnets, acquiring magnetic polarity during their construction, and further fields are induced as the ship moves about. The ship's magnetic field had to be measured and countered electromagnetically. Depending on the vessel, this could require from one to three sets of copper coils, and occasionally more. The coils ran so hot that a pinhole in the insulation of 1-inch cables would send molten copper spraying out! By 1943, such a large percentage of the nation's copper production was used for making insulated 1-inch cables for ship degaussing, that the US Government was manufacturing pennies from steel, not copper.

This was a stressful and heady atmosphere, but one which Ralph seemed to take in stride. His personnel evaluations demonstrate this. He was a physics researcher by day, and GWU physics student by night. Coffee and cigarettes were rationed beginning in 1942—staples of any graduate student's diet at the time. Reused coffee grounds made weak coffee but were essential. Instant coffee was pioneered by General Foods in 1941 for combat soldiers—but Postum[TM] was a popular substitute. Following the War, Ralph continued to crave thick black (motor oil) coffee for the rest of his life.

One research project to test the complicated degaussing systems involved Ralph's travel aboard the *USS Massachusetts* from Norfolk, VA, to New York harbor (which

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had been mined by U-Boats). Even with the use of minesweepers, this was dangerous applied physics!

Ralph also worked on Naval torpedo guidance and detonation systems. One research expedition took him to Puget Sound. From a wooden rowboat, he and his colleagues evaluated the trajectory of water-borne torpedoes. The torpedoes "broached" wildly out of the water frequently enough to result in major dousings in salt water. There could be no question that this was an exciting time to be an applied and experimental scientist. Results were a priority and there were seemingly unlimited funds. Nonetheless, in absolute terms, the total amount spent on such research and development amounted to only two or three days' worth of Government expenditure on the war in 1945.

Ralph also was involved in anti-submarine warfare, much of which was secret and remains highly classified today. The Navy Department summaries at the end of the war (which were secret, numbered, and of limited distribution) briefly described a program that Ralph explained to me, called MAD (magnetic airborne detector) which was designed to detect U-boats at periscope depth, from which the large majority of torpedo attacks were mounted. This method led to airborne bombing and depth-charging methods efficient enough to significantly diminish the effectiveness of U-boats.[4, 5]

Despite his service in defense-related research, Ralph Alpher was called up for a physical for the draft in 1944. On his draft notice, he listed his skills—that he had learned to operate professional opticians' equipment (two of his Russian uncles ran an optician's shop). He also included typing at 100 words per minute, Gregg Shorthand, and physics. He was not drafted. Interestingly, his Russian ancestry became the subject of a fairly intense FBI investigation for his security clearance in the early 1950s.

Mystery in the Sky—the Secret Fuse

Prior to World War II, anti-aircraft guns could only fire time-delay and contact-detonated shells. It was very difficult to strike a moving airplane. Needed was a projectile that would sense only its proximity to a moving target to detonate.

The details of the Proximity Fuse were secret—the greater picture only declassified substantially by 1976.[6] A prewar idea, it took the team of scientists, engineers, and radio amateurs at Section T to make a Proximity Fuse work, even though the British had worked feverishly on the concept until 1940. Those were the days before transistors. Radios were operated by vacuum tubes, and for a radio signal to operate in a large projectile such as an anti-aircraft shell, it had to withstand high *g*-forces (20,000 *g*) and up to 48,000 rpm in the muzzle of a 5-inch anti-aircraft gun aboard ship. This is not a healthy environment for vacuum tubes.

Parts of the VT Fuse were designed, tested, and manufactured by electronics manufacturers across the nation (such as Crosley and Raytheon), but were distributed and assembled in such a way that no one ever figured out what it was they were making for the Government. During World War II, no one asked, because it was a matter of national security of a degree that few remember or understand today. Workers at Section T could not tell their wives what they were working on-no doubt straining many young marriages maximally.[7]

Section T surmounted these obstacles—and over a relatively short period finally developed a vacuum-tube-operated radio sender and receiver that could explode and disperse shrapnel close to the reflecting source. At 75 to 100 feet, this is close enough to blast a fighter or dive-bomber out of the sky. The increase in effectiveness over conventional contact or time-delay fuses was multifold. There had been no real advances in this technology since before the American Civil War. Although the British and Americans also had developed photoelectric fuses, their dependence on sunlight favored the Proximity Fuse, which operated day or night.

The VT Fuse finally met Navy reliability tests in 1942 (50 percent). There were some 300 parts to each mechanism, and they had to work to perfection. Even a 99 percent reliability rate in components will yield only a four percent success rate—far below the 50 percent rate required for the Navy to continue funding the work. When the ultimate and most crucial tests were made, three US Army Air Force drone airplanes were destroyed in the air near the Potomac.

Fortunately, the rationing of coffee had ended in late 1943, just in time for Section T to begin working on the many applications of the VT Fuse. The VT Fuse made its operational debut in the Battle of Midway on January 9, 1943. On some documentaries today, one can see interviews of once-bewildered Japanese pilots who had for the first time seen planes—even Kamikazes—dropping from the sky without being hit directly by an anti-aircraft shell. When used during the Battle

The Applied Physics Laboratory at 8621 Georgia Avenue in Silver Spring, Maryland, converted from an automobile dealership. Note there is no sign to suggest what this building is being used for—an exception during wartime had to be obtained for any new construction or remodeling of existing structures—and the uniformity of vehicles parked around the building. After World War II, this became the Applied Physics Laboratory of Johns Hopkins University.

(From the personal photo collection of Ralph A. Alpher; courtesy of Victor S. Alpher).

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of the Phillippine Sea, the VT Fuse was so effective against enemy aircraft and ships that the Battle became known in the Navy as a "Turkey Shoot." The Army named the fuse the Pozit fuse, after the word "posit," meaning "to position" a shell at a particular place. After the war, it was found that Germany had tried to reconstruct a recovered VT-fused shell, but their scientists could not figure out how it operated.[8] Japan finally created one, but after its first use, the US Army Air Force destroyed the Japanese production facilities and it was never seen again in action.

It has been suggested that many of the German V-1 and V-2 rockets, which rained down on London during the Blitz of late 1944, met their demise in the vicinity of counterforces armed with the Proximity Fuse, which were moved closer to the southern coast of England. The Yanks saved England again! Used in 81 mm mortar shells during the Battle of the Bulge, Posit-fused shells were extremely effective against unsheltered German troops and were instrumental in bringing the brief German counter-offensive of late 1944 to a close.[9] General George S. Patton ordered as many of the "funny fuses" as he could possibly obtain—predicting that their effectiveness would lead to new tactics in any future war. Ralph and the other scientists and technicians at 8621 Georgia Avenue received these reports with a combination of excitement and equanimity. Their success was fortuitous—yet lamentably destructive.

Ralph continued progress towards his Master's thesis in 1945—even while making a detour to work within the Manhattan Project. He made at least one journey to the high desert of New Mexico at Los Alamos. There, he helped determine the flight path of the *Enola Gay*—so that it would escape the radiation and percussive force of the atomic blast.

At the end of the war, the administration of Section T was handed over to Johns Hopkins University (JHU) and became the JHU Applied Physics Laboratory (APL), the first of its kind in private hands. JHU-APL effectively became the first private technical research laboratory producing useful advanced technology for the public and private sectors.

Remade German V-2 unmanned rocket such as the type used in the second London Blitz in 1944, prior to test firing in May 1946 in New Mexico. The Proximity Fuze proved essential to the defense of Britain against the V-1 and V-2 bombs created by the Germans under the leadership of Wernher von Braun, who ultimately became the head of the Marshall Space Flight Center and created the Saturn V rocket, which took Americans to the moon. At this time, former German scientists were working at Fort Bliss, Texas, and launching test rockets at White Sands Proving Ground in New Mexico.

(From personal photo collection of Ralph A. Alpher; courtesy of Victor S. Alpher).

Do Not Underrate Night School: Ralph Alpher is Famous at 27!

Ralph remained with the JHU-APL for another nine years following the war, until 1955, turning his efforts after the war to the physics of airflow around missiles. Section T no longer existed, but JHU-APL endured and prospered. Most of the work continued to be defense-related, but Ralph's scientific life had taken a significant turn under George Gamow's tutelage.

Before becoming a doctoral student, Ralph had translated an important 1943 German technical work into English,[10] and before finishing his thesis, he passed fluency tests in German and French (I still have his Cassell's 1939 German dictionary—in Old German Fraktur script)! Many schools today no longer require fluency in two languages for a doctoral degree; in the mid-20th century, this was considered a hurdle to be overcome by all and essential to the life of a scholar.

In 1948, Ralph was still in night school at GWU. He had to ask for 24 days of work leave to prepare for his comprehensive doctoral examinations. How did he manage this pace of work in an era with no cell phones, no fax machines, no 4-function calculators, no Internet, no personal computers or Macs—with nothing but imagination, K & E slide rules, a library, No. 2 pencils, graph paper, and carbon copies?

Ralph's dissertation, defended in early 1948, broke the code—theoretically—of the creation and distribution of elements during the first minutes following a presumed "Big Bang." The announcement of Ralph's dissertation defense in the GWU newspaper—a common event on university campuses across America, yielded an extraordinary response in the community, including an appearance on television.

His defense was a media event. In order to house the dissertation defense, a room that could accommodate 300 people was required. Many were from the press, and the defense was held in full academic regalia—rarely if ever seen today! *HERBLOCK*, a famous newspaper political cartoon for several decades (by Pulitzer Prize-winning cartoonist Herb Lock), devoted one of his daily cartoons to the subject. What appears

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to be a large bomb is sitting on a club chair, reading a newspaper with the large headline "Scientist Says World Was Created in Five Minutes." Scratching its chin, the bomb furls its eyebrows and thinks, "Five Minutes, Eh?" He is pondering a statement from Ralph's dissertation that implies that the period of nucleosynthesis in the early universe lasted about five minutes.

In his dissertation, a series of calculations suggested that 90 percent of the elements, helium and hydrogen, were created within about the first 17 seconds after the Big Bang. The "neutron-capture" theory in the expansion of the universe from a hot, dense, singularity demonstrated that the elements were formed by the "capture" of protons by neutrons; and that after an atomic weight of 3 the elements were created more slowly; and that elements with an atomic weight greater than 250 had probably disappeared. His dissertation contained a series of 89 hand-drawn equations and calculations, and was only 74 pages long.[11]

Ralph's work demonstrated that cosmology and astrophysics had entered the modern era of physics, and that the "Big Bang"—once an idea—could be demonstrated. Although it is not uncommon for thesis advisors to have their names included in dissertation publications, George Gamow, a reknowned prankster, added Hans Bethe's name to the very first publication of Ralph's findings—attributed to Alpher, Bethe, and Gamow (alpha, beta, gamma of the Greek alphabet). The story of the creation of the elements in the early Universe first appeared as a letter in Nature on April Fool's Day (April 1), 1948.

Those days were turbulent times—the beginning of the Cold War, the Berlin Airlift, controversy about the military uses of atomic physics, fears of global destruction amidst economic boom, and rebuilding of Europe under the Marshall Plan. So, news of the "Big Bang" at that time had a short media life. Until the launching of Sputnik, physics was considered mainly a defense-related science by many.

Faint Echoes of the Big Bang

Shortly after his dissertation publication, with Robert C. Herman on board, Ralph came up with a prediction of the Cosmic Microwave Background Radiation (CMBR), the distant cooling echo of the "Big Bang" at merely a few degrees Kelvin—but they could not find a radio astronomer who believed the prediction could be confirmed! Nonetheless, confirmation of the CMBR in 1964 by radio telescope would lead to four Nobel Prizes (1978 and 2006). Gradual acceptance as the "Big Bang" became part of common parlance. However, the seminal work of Ralph Asher Alpher was still largely ignored—a subject of continued controversy and personal soul-searching.[12, 13, 14]

George Gamow passed away in 1967—having seen the empirical vindication of Ralph's work. Robert C. Herman died in 1997, having experienced some recognition within the physics community of the significance of his work with Ralph. However, recognition of Ralph Asher Alpher's singular contributions had to wait until weeks before he passed away. On July 27, 2007, he was awarded the 2005 National Medal of Science.[15]

The Big Bang to be continued...

Acknowledgments

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Composite photo showing George Gamow emerging from a bottle of "YLEM" (primordial "stuff" of the Universe), produced in the late 1940s. Ralph A. Alpher and Robert Herman flank the bottle at the right and left, respectively.

(From the personal photo collection of Ralph A. Alpher; courtesy of Victor S. Alpher).

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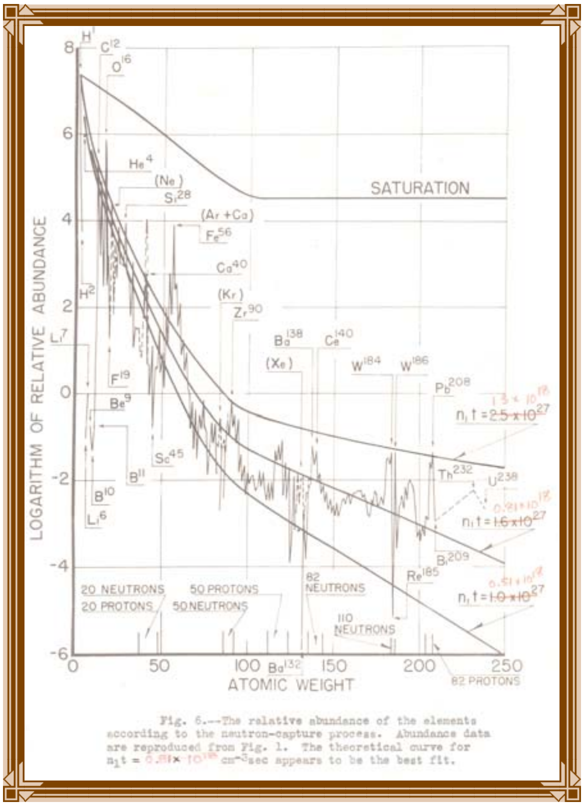

Graph from Ralph A. Alpher's doctoral dissertation, 1948[15], showing "best fit" curves to available data on abundance of the chemical elements. The expansion (Big Bang) theory held that the most abundant elements, formed by the neutron-capture process, occurred by about 17 seconds after the moment of singularity.

In order to avoid confusion amongst the expansion model and equilibrium models, Alpher proposed the term "saturation" for the point in time when most elements had been formed. He stated that the expansion model works when expansion (the Big Bang) "started from a point of singularity of infinite density"[16]. At saturation (above atomic weight 250, approximately) heavier elements continue to decay and disappear—and hence, we don't know of them. Figure 6 on page 61 of the dissertation is labeled "The relative abundance of the elements according to the neutron-capture process. Abundance data are reproduced from Fig. 1. The Theoretical curve for n1t = 0.81 x 1018 cm-3 sec appears to be the best fit."

(From the personal photo collection of Ralph A. Alpher; courtesy of Victor S. Alpher.)


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We see that Eqs. (69) are equivalent to $\lambda_j n_j$ in Eqs. (60), so that

(71)

With the forms assumed for σ in Eqs. (68), it is possible to complete the evaluation of the quantities λ_j. Thus,

(72)

and, similarly, for j < 100,

(73)

For j > 100, the factor $\sqrt{(i+j)/j}$ differs very little from unity, so that Eq. (72) can be written

(74)

Temperature has cancelled out as a parameter in the equations of the building-up process, as a result of the forms assumed for σ. However, there are several factors which must have essentially limited the temperature at which the building-up process went on. Firstly, as already mentioned, the abundance plot does not show, generally, marked deviations corresponding to those elements possessing abnormally large capture resonances for neutrons. It seems reasonable to suppose that the temperatures during the

Page 55 from Ralph Alpher's dissertation (carbon copy in his personal papers) shows hand-drawn equations drawn in Pelikan prewar ink, and red-pencil corrections which he noted on another page were corrected in the dissertations submitted to the Graduate Council. This page comes from Chapter VI entitled: "The Neutron-Capture Theory of the Formation and Relative Abundance of the Elements."

(From the personal photo collection of Ralph A. Alpher; courtesy of Victor S. Alpher.)

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[15] The National Medal of Science "recognizes those who have made outstanding contributions to knowledge in the sciences." The citation for the 2005 Award, made in July 2007, recognized Ralph A. Alpher "[f]or his unprecedented work in the areas of nucleosynthesis, for the prediction that universe expansion leaves behind background radiation, and for providing the model for the Big Bang theory."
[16] Ref. 11, p. 24.