On the Electrodynamics of Moving Bodies (Part A: Kinematics) by Albert Einstein

ELEGANT CONNECTIONS IN PHYSICS

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James Clerk Maxwell observed in *A Treatise on Electricity and Magnetism*, "It is of great advantage to the student of any subject to read the original memoirs on that subject, for science is always most completely assimilated when it is in its nascent state." The mathematician Niels Abel, when asked how he could accomplish so much by age 27, said "By studying the masters, not the pupils." As physics students we studied the Special Theory of Relativity. Yet Albert Einstein's original paper that started it all, "On the Electrodynamics of Moving Bodies,"[1] seldom appears on our course reading lists. In the hope of making Einstein's great paper more accessible, in this annotation I attempt to express Einstein's insights in familiar notation, and fill in some intermediate steps between his postulates and results. For best results, read Einstein's paper[2,3] and consult these notes as necessary, as together we enter the mind of Einstein.

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Albert Einstein began "On the Electrodynamics of Moving Bodies"[1] by noting a puzzling equivalence between electric and magnetic forces:

It is well known that Maxwell's electrodynamics—as usually understood at present—when applied to moving bodies, lead to asymmetries that do not seem to be inherent in the phenomena.[2,3]

Einstein illustrated the "asymmetry" with a magnet and a conductor (Fig. 1). An observer at rest *relative to a conducting loop* sees a changing flux of the magnetic field $\mathbf{B}$ as the magnet glides through the loop. Faraday's law induces an electric field $\mathbf{E}$, which drives a current in the loop by the *electric* force $q\mathbf{E}$ acting on particles of charge q. But an observer at rest *relative to the magnet* sees the loop sweep by with velocity $\mathbf{v}$; now the *magnetic* force $q\mathbf{v}\times\mathbf{B}$ drives the current. Identical *results* are produced by seemingly different mechanisms distinguished by relative motion.
What explains this serendipitous equivalence?—which begs a deeper question: What is the relationship between electric and magnetic fields and the motion of the observer relative to them? To explain this mystery, Einstein laid down a new conceptual framework by announcing two postulates:

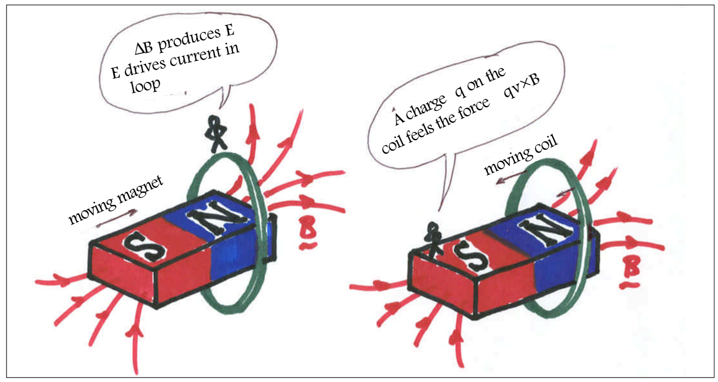

Fig. 1: Depending on the reference frame, the current in the loop is driven by either (a) an electric force, or (b) a magnetic force.

Examples of this sort, together with the unsuccessful attempts to detect a motion of the earth relative to the 'light medium,' lead to the conjecture that not only the phenomena of mechanics but also those of electrodynamics have no properties that correspond to the notion of absolute rest. Rather, the same laws of electrodynamics and optics will be valid for all coordinate systems in which the equations of mechanics hold good...We shall raise this conjecture (whose content will hereafter be called the "principle of relativity") to the status of a postulate and shall also introduce another postulate, which is only seemingly incompatible with it, namely that light always propagates in empty space with a definite velocity c that is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent electrodynamics of moving bodies...

The speed of light—a unique velocity scale whose value $c = 3\times10^8$ m/s in vacuum emerges from Maxwell's electrodynamics—equals a ratio of distance to time. Einstein's "relativity of electrodynamics" program implies a theory of space and time—which had been the subjects of philosophical argument for centuries.[4] When Newton introduced mechanics in the *Principia* he stated "I take space to be absolute, I take time to be absolute." By that, he meant space is everywhere the same, so the length of a rigid rod would be the same for all observers, regardless of their motion relative to the rod; and likewise, the time interval between two events would also be the same for all observers. This point of view was criticized

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by Leibniz who saw space and time, not as *things* in themselves, but as *relations* between observer and observed. Einstein bypassed such arguments and asked basic *physics* questions, about how one *actually measures* the time between separated events, and the lengths of moving bodies.

Like all electrodynamics, the theory to be developed here is based on the kinematics of a rigid body, since the assertions of any such theory have to do with the relations among rigid bodies (coordinate systems), clocks, and electromagnetic processes. Insufficient regard for this circumstance is at the root of the difficulties with which the electrodynamics of moving bodies currently has to contend.

Following this introduction, in Section A of his paper, Einstein derives a new kinematics that follows from his two postulates. Throughout the paper he refers to two inertial frames as the "rest system" and the "moving system." We find him speaking of a "rod at rest in the moving system," and a "rod moving through the rest system." For clarity of visualization I will employ Taylor and Wheeler's images of the "Lab frame" and the "Rocket frame."[5] Both are inertial frames; the Rocket moves at constant velocity $\mathbf{v}_R$ relative to the Lab. The Lab frame observer records events in space and time with unprimed coordinates (t, x, y, z); the Rocket frame employs primed coordinates (t', x', y', z'). The respective axes such as x and x' are parallel; the clocks throughout both frames read zero at the instant when the two origins coincide; and the Rocket moves relative to the Lab in a direction parallel to the Lab's x axis. In the Rocket frame the Lab moves parallel to the x' axis with velocity $-v_R$.

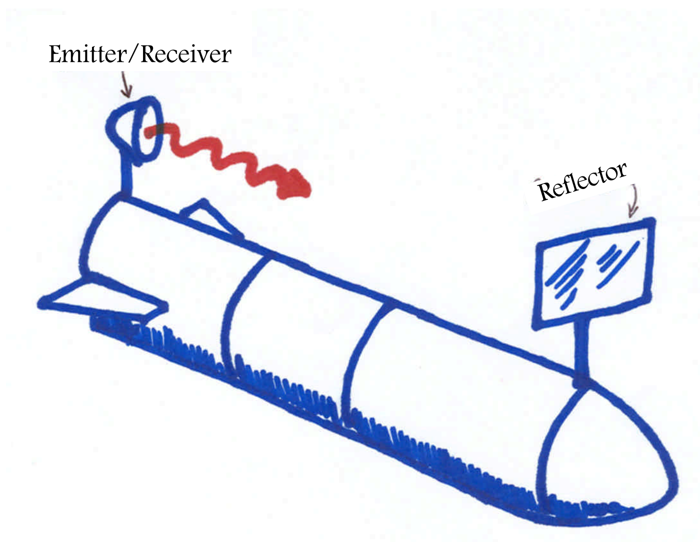

Fig. 2. The apparatus used for Event 1 (emission of light), Event 2 (reflection), and Event 3 (detection). All apparatus is carried aboard the Rocket Frame, but Events one, two, and three will be observed from both the Rocket and Lab Frames.

A. Kinematic Part

1. Definition of Simultaneity

2. On the Relativity of Length and Time

Einstein reminds us that to describe the motion of a particle, we specify its position coordinates as functions of time. But he then draws our attention to a simple but under-appreciated fact:

...We have to bear in mind that our judgments involving time are always judgments about simultaneous events. If, for example, I say that 'the train arrives here at 7 o'clock,' that means, more or less, 'the pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events.'

This realization of the pivotal role of simultaneity started the Special Theory of Relativity. Because the watch can be located at one place only at any one instant, we encounter difficulties in using Einstein's pocket watch to measure time throughout *all* space, as Newton assumed could be done with his paradigm of absolute space and time, and their presumed instantaneous communication. But due to the *finite* speed of light, Einstein realized that such a definition of time *"is no longer satisfactory when series of events... occurring at places remote from the clock have to be evaluated temporally."* If we were to post an observer at the origin with the pocket watch, we could define the time of a remote event (such as the emission of light by the Sun) as the reading of the pocket watch when a light signal from that event reaches our watch-bearing observer (when the observer sees the light). Clearly, if the speed of light is finite, the emission and reception are separated by a finite time interval. And that would make our understanding of all events dependent on the position of one watch. *"We reach a far more practical arrangement by the following argument."*

Distribute clocks throughout the reference frame, suggests Einstein. *"If there is a clock at point A in space, then an observer located at A can evaluate the time of events at A..."* just as Einstein measured the train's time of arrival locally. Likewise, with another clock at B the time of events that occur there can also be measured locally. *"But it is not possible to compare the time of an event at A with one at B without further stipulation."* The clocks must be synchronized, as Einstein describes in the following thought experiment.

Let's fasten our points *A* and *B* to the Rocket frame. Bolt a light source and a detector, along with a clock, to the tail of

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the Rocket; and bolt a mirror with another clock to the rocket's nose (see Fig. 2). Let a light pulse be *emitted* from the tail (event 1), let the pulse *reflect* from the mirror on the nose (event 2), and let the pulse be *detected* back at the rocket's tail (event 3). Now data can be gathered by this array of observers, each one at rest relative to the Rocket, using a *local* clock infinitesimally close to an event of interest. The time required for the light to go from event 1 to event 2 equals the time for light to go from event 2 to event 3:

$$t'_2 - t'_1 = t'_3 - t'_2 \,. \tag{1}$$

If we let $t'_1 = 0$, then $t'_2 = L'/c$, where L' denotes the length of the rocket in the Rocket Frame. Note that $t'_3 = 2L'/c = 2\,t'_2$. With synchronized clocks, when the clock at the detection event reads $2L'/c$, the clock at the mirror *simultaneously* reads $2L'/c$ as well. The arrival of the light at the detector, and the clocks located at the mirror and the detector reading $2L'/c$, are simultaneous events.

But now look at events one, two, and three from the *Lab* frame. The Lab observer sees the rocket of length L zoom by in the x-direction with velocity v_R, carrying the light source, mirror, and detector with it. According to the second postulate, the speed of the light pulse must be c in the Lab frame, as it was in the Rocket frame.

Let the emission event as recorded in the Lab frame occur at the origin at time t_1. When event 2 occurs, the mirror is located in the Lab Frame at $x_2 = L + v_R(t_2 - t_1) = c(t_2 - t_1)$, so that

$$t_2 - t_1 = L/(c - v_R). \tag{2}$$

In the time required for the light to make the trip from the mirror to the detector, the detector moves to the right in the Lab Frame the distance $v_R(t_3 - t_2)$. So between Events 2 and 3 the light travels the distance $c(t_3 - t_2) = L - v_R(t_3 - t_2)$, and thus

$$t_3 - t_2 = L/(c + v_R)\,. \tag{3}$$

Clearly, $t_2 - t_1 \neq t_3 - t_2$. If we set $t_1 = 0$, then $t_2 = (L/c)(1 - v_R/c)^{-1}$ and $t_3 = (2L/c)(1 - v_R^2/c^2)^{-1} < 2t_2$. Unlike the situation in the Rocket frame where $t'_3 = 2t'_2$, in the Lab frame $t_3 \neq 2t_2$. *Simultaneity of two events is not an invariant* between reference frames. If the time intervals between events are *not* invariant, how *are* they related?

3. Theory of Transformations of Coordinates and Time from the Rest System to a System in Uniform Translational Motion Relative to It

Einstein begins this section laying out the two inertial coordinate systems that we have already introduced as the Lab and Rocket frames.[6] He now must relate the x, y, z, and t coordinates of an event in the Lab frame to its coordinates x', y', z', and t' in the Rocket frame. Always one to use symmetry to the hilt, Einstein remarks, *"First of all, it is clear that these equations must be linear because of the properties of homogeneity that we attribute to space and time."*

Consider the Events 1, 2, and 3 that were discussed in the preceding section. In the Rocket frame, Event 2 occurred halfway in time between Events 1 and 3,

$$t'_2 = \tfrac{1}{2}\,[t'_1 + t'_3]\,, \tag{4}$$

where, in the Lab frame, by Eqs. (2) and (3),

$$t_2 = t_1 + L/(c - v_R)$$

and

$$t_3 = t_1 + L/(c - v_R) + L/(c + v_R) \tag{5}$$

For each of these three events, Einstein will write its t' coordinate in Eq. (4) as functions of the Lab coordinates x, y, z, and t. A point at rest in the Rocket frame (imagine a dot painted on the rocket) glides through the Lab frame with speed v_R. Let the dot's coordinate in the Lab frame be x at time t; then, for that dot, the quantity $X \equiv x - v_R t$ is time-independent. Therefore t' may be written as the function $t' = t'(X, y, z, t)$.

When Event 1 (emission) occurs, let the Lab clocks read time t_1 and let the Rocket origin's location in the Lab frame be $x_1 = v_R t_1$. It will be noticed that, for Event 1, $X_1 = 0$. Event 2 (reflection) occurs for some non-zero value of X. Event 3 (detection) occurs at the Rocket frame's origin where $X_3 = 0$. Eq. (4) becomes

$$t'[X,0,0,\, t + X/(c - v_R)]$$
$$= \tfrac{1}{2}t'[0,0,0,t] + \tfrac{1}{2}t'[0,0,0,t + X/(c - v_R) + X/(c + v_R).] \tag{6}$$

Einstein expands each t' in a Taylor series about $X = 0$. Suppressing the arguments for y and z, such a Taylor series looks like this:

$$t'(X,T(X)) = t'(0,0) + X(dt'/dX)_0 + X(dt'/dT)_0(dT/dX)_0 + \ldots \tag{7}$$

With $T = t + X/v_R$, Eq. (7) becomes

$$t'(X,\, t{+}X/v_R) \approx t'(0,0) + X(dt'/dX) + (X/v_R)(dt'/dt)\,. \tag{8}$$

Using Eq. (8), the surviving terms in Eq. (6) are

$$(dt'/dX) + v_R(c^2 - v_R^2)^{-1}\,(dt'/dt) = 0 \tag{9}$$

which can be solved by separation of variables. Let

$$dt'/dt = \varphi = const\,.$$

then

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$dt'/dX = -\varphi\ v_R(c^2 - v_R^2)^{-1}$.

where φ depends on v_R. These integrate to $t' = \varphi t + const.$ and $t' = -\varphi X v_R(c^2 - v_R^2)^{-1} + const.$, which together yield

$$t' = \varphi[t - v_R X (c^2 - v_R^2)^{-1}] . \quad (10)$$

The additive constant vanishes if $t' = 0$ and $x = 0$ when $t = 0$. A light ray emitted from the Rocket origin at time $t' = 0$ arrives later at the location $x' = ct'$, that with Eq. (10) gives

$$x' = \varphi\, c\, [t - v_R X(c^2 - v_R^2)^{-1}] . \quad (11)$$

But as measured in the Lab frame, *"the light ray propagates with velocity c – v_R relative to the origin of the [Rocket frame]."* In light of the Second Postulate, I had to do a double-take on this statement. Here is what Einstein means: Consider a race between the light ray and the rocket, as viewed from the Lab frame. Let the racers cross the starting line together at $t = 0$. The velocity of the light ray exceeds that of the rocket's *origin* by $c - v_R$, as seen *from the Lab*. For times $t > 0$ the distance between the light ray and the rocket origin is $(c - v_R)t$. If this is made equal to the distance in the Lab frame between the emission and reflection events then $X = (c - v_R)t$, and Eq. (11) becomes

$$x' = \varphi\, c^2\, X\, (c^2 - v_R^2)^{-1} . \quad (12)$$

With Einstein we now turn to the relativity of the spatial dimensions that are orthogonal to the relative motion between the Lab and Rocket frames. Consider a light pulse emitted from the origin of the Rocket (when coincident with the Lab origin) and sent along the y' axis. If the light hits another mirror at $y' = ct'$ then Eq. (10) gives

$$y' = \varphi\, c[t - v_R X\, (c^2 - v_R^2)^{-1}] . \quad (13)$$

For this particular reflection event $X = 0$ since it's Lab x-coordinate equals $v_R t$. Einstein also notes that, as seen from the Lab frame, this *"light always propagates along [the y and z axes] with the velocity $(c^2 - v_R^2)^{½}$,"* which requires (at least for me) another double-take. Einstein seems to be envisioning this situation: Let a spherical pulse of light be emitted from the Lab frame origin, and consider the reception of this light pulse by a mirror carried aboard the Rocket. The pulse bounces off this mirror at Lab frame coordinates $x = v_R t$ and $y = v_y t$. Therefore, a point on this spherical light wave has traveled, in the Lab frame, the distance

$$ct = [(v_R t)^2 + (v_y t)^2\,]^{½} . \quad (14)$$

Thus $v_y = (c^2 - v_R^2)^{½}$ denotes the speed, relative to the Lab, with which the light pulse slides up the y'-axis. Because in the Lab frame the y-coordinate of the light pulse is $y = v_y t$, Eq. (13) becomes

$$y' = \varphi\, cy/v_y = \varphi\, cy\, (c^2 - v_R^2)^{-½} . \quad (15)$$

Likewise,

$$z' = \varphi\, cz\, (c^2 - v_R^2)^{-½} . \quad (16)$$

If we recall that, in general, $X = x - v_R t$ then our results so far say that

$$t' = \varphi\, \gamma (t - v_R x/c^2) , \quad (17a)$$
$$x' = \varphi\, \gamma (x - v_R t) , \quad (17b)$$
$$y' = \varphi\, y , \quad (17c)$$
$$z' = \varphi\, z \quad (17d)$$

where

$$\gamma \equiv (1 - v_R^2/c^2\,)^{-½} \quad (18)$$

with φ still an unknown function of v_R. Before determining φ Einstein performs a consistency check:

Now we have to prove that, measured in the [Rocket frame], every light ray propagates with the velocity c, if it does so, as we have assumed, in the [Lab frame]; for we have not yet proved that the principle of the constancy of the velocity of light is compatible with the relativity principle.

He imagines a spherical light wave to be emitted from the instantaneously coincident coordinate origins of the Lab and Rocket frames, an event for which the clocks in both frames read $t = 0$ and $t' = 0$. At some later time, an arbitrary point on the spherical wave front in the Lab frame is given by $x^2 + y^2 + z^2 = c^2\, t^2$.

We transform this equation using our transformation equations and, after a simple calculation, obtain

$$x'^2 + y'^2 + z'^2 = c^2\, t'^2 .$$

Thus, our wave is also a spherical wave with propagation velocity c when it is observed in the [Rocket frame]. This proves that our two fundamental principles are compatible.

Students of relativity will celebrate this first appearance of the "Spacetime Interval."

To determine φ Einstein now introduces a third reference frame, called here the Rocket-Prime frame (with double-primed coordinates), which moves parallel to the x' axis of the Rocket frame with velocity $-v_R$ relative to the Rocket. Let all three origins coincide and let their clocks all read 0 at a certain event. By twofold application of his transformation equations, Einstein writes for a subsequent event,

$$t'' = \varphi\, (-v_R)\ \gamma (t' + v_R x'/c^2) = \varphi(-v_R)\ \varphi\, (v_R) t \quad (20)$$

$$x'' = \varphi\, (-v_R)\ \gamma (x' + v_R t') = \varphi\, (-v_R)\ \ \varphi\, (v_R) x \quad (21)$$

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$$y'' = \varphi(-v_R)\,\varphi(v_R)y, \quad z'' = \varphi(-v_R)\,\varphi(v_R)z \tag{22}$$

But Rocket-Prime is at rest relative to the Lab; hence

$$\varphi(-v_R)\,\varphi(v_R) = 1\ . \tag{23}$$

Next, consider a rod of length L' carried aboard the rocket and oriented along the y' axis. According to Eqs. (17), its endpoints in the Lab frame are given by $(x, y, z) = (v_R t, L/\varphi, 0)$ for the upper end, and $(v_R t, 0, 0)$ for the lower end. By symmetry, *"the length of the moving rod measured in the [Lab frame] does not change if v_R is replaced by $-v_R$"* so it follows that $\varphi(-v_R) = \varphi(v_R)$. Together with Eq. (23) this gives $\varphi(v_R) = 1$. The transformations of Eqs. (17) are therefore

$$t' = \gamma\,(t - v_R x/c^2) \tag{24a}$$
$$x' = \gamma(x - v_R t) \tag{24b}$$
$$y' = y \tag{24c}$$
$$z' = z \tag{24d}$$

This is the "Lorentz transformation," which had been introduced *ad hoc* by Lorentz in 1902 to explain the null result of the Michelson-Morley experiment.[7] It emerges as a *consequence* of Einstein's postulates.

4. The Physical Meaning of the Equations Obtained as Concerns Moving Rigid Bodies and Moving Clocks

To gain insight into the implications of his results derived so far, Einstein considers a rigid body sphere of radius R at rest in the Rocket frame and centered on its origin. In this frame the equation of the sphere's surface is

$$x'^2 + y'^2 + z'^2 = R^2\ . \tag{25}$$

By the Lorentz transformation Eq. (25) becomes

$$x^2\,(1 - v_R^2/c^2)^{-1} + y^2 + z^2 = R^2\ . \tag{26}$$

This shape, expressed now in Lab coordinates, is that of an ellipsoid of revolution, having axes of length

$$R\,(1 - v_R^2/c^2)^{-½},\ R,\ R\ .$$

Unlike the spherical pulse of light, a rigid body sphere does *not* maintain its shape between the Lab and Rocket frames:

"Thus, while the y and z dimensions of the sphere...do not appear to be altered by motion, the x dimension appears to be contracted in the ratio 1: $(1 - v_R^2/c^2)^{-½}$, thus the greater the value of v_R, the greater the contraction. For $v_R = c$, all moving objects—considered from the [Lab frame]—shrink into plane structures. For superluminal velocities our considerations become meaningless; as we shall see from later considerations, in our theory the velocity of light physically plays the role of infinitely great velocities."

This was our first glimpse of length contraction in Einstein's conception of space and time, and the first pronouncement that the speed of light serves as the ultimate "speed limit" in the universe.

Einstein next applies these results to derive the relativity of time. Considering a clock carried aboard the Rocket, he asks *"What is the rate of this clock when considered from the [Lab frame]?"* The Lab coordinates of an event, and its time t' as recorded in the Rocket clocks, are connected by Eqs. (24),

$$t' = (1 - v_R^2/c^2)^{-½}\,(t - v_R x/c^2) \tag{27}$$

where in the Lab frame $x = v_R t$, turning Eq. (27) into

$$t' = t\,(1 - v_R^2/c^2)^{½}\ . \tag{28}$$

Students of Special Relativity will recognize Eq. (28) as the "time dilation formula." In the situation considered here, t' denotes the time elapsed on the clock bolted to the dashboard of the rocket. Thus, two "ticks" of this clock occur, in the Rocket frame, at the same place, and thus measure the "proper time" of these events, as we say today. Einstein did not introduce the term "proper time" here, but looking at his formula he added and subtracted t to emphasize the time *difference*

$$t' = t - t[1 - (1 - v_R^2/c^2)^{½}]\ , \tag{29}$$

which we may write as $t - t' = \delta$, where $\delta = t[1 - (1 - v_R^2/c^2)^{½}] > 0$. This means that t reads more time than does t' for the same two events; or to say it with Einstein's phrase, the Lab clock "lags behind" the Rocket clock by the amount δ. This led Einstein to an astonishing consequence:

"..If there are two synchronously running clocks at A, and one of them is moved along a closed curve with constant velocity until it has returned to A, which takes, say, t sec, then, on its arrival at A, this clock will lag $½t(v_R/c)^2$ sec [to lowest order in v_R/c] behind the clock that has not been moved. From this we conclude that a balance-wheel clock located at the Earth's equator must, under otherwise identical conditions, run more slowly by a very small amount than an absolutely identical clock located at one of the Earth's poles."

There are legion experimental demonstrations of time dilation, such as the ubiquitous muons-in-cosmic-rays example that appears in all the textbooks. When time could be measured to nanosecond precision fifty years after Einstein wrote these preceding lines, an experiment was done that recalled Einstein's prediction explicitly:

The experiment was done by a young man called H.J. Hay at Harwell. He imagined the earth squashed flat into a

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plate, so that the North Pole is at the centre and the equator runs round the rim. He put a radio-active clock on the rim and another at the center of the plate and let it turn. The clocks measure time statistically by counting the number of radio-active atoms that decay. And sure enough, the clock at the rim of Hay's plate keeps time more slowly than the clock at the centre. This goes on in every spinning plate, on every turntable. At this moment, in every revolving gramophone disc, the centre is ageing faster than the rim with every turn.[8]

5. The Addition Theorem for Velocities

Let a particle moving relative to the Rocket be described by the coordinates

$$x' = w_1 t', \; y' = w_2 t', \; z' = 0 \qquad (30)$$

where w_1 and w_2 are constant velocity components for the particle's velocity relative to the Rocket. Einstein seeks the velocity of this particle relative to the Lab frame. With Eqs. (24) and some algebra, Eqs. (30) become, in Lab coordinates

$$x = t(w_1 + v_R) / (1 + w_1 v_R/c^2) \, , \qquad (31)$$

$$y = w_2 \, \gamma (t - v_R x/c^2) \, , \qquad (32)$$

and $z = 0$. Using Eq. (31) to eliminate x in Eq. (32), y can be written as a function of t:

$$y = w_2 t \, (1 - v_R/c^2)^{1/2} \, (1 + w_1 v_R/c^2)^{-1} \, . \qquad (33)$$

From the definition of velocity, the particle's velocity v in the Lab frame can be calculated from

$$v^2 = (dx/dt)^2 + (dy/dt)^2 \qquad (34)$$

while its velocity w in the Rocket frame is

$$w^2 = w_1^2 + w_2^2 \, . \qquad (35)$$

Also, let

$$\alpha = \arctan (w_2/w_1) \, . \qquad (36)$$

Eqs. (31) and (33-36) together turn v in Eq. (34) into

$$v = \frac{[v_R^2 + w^2 + 2 v_R w \cos \alpha - (v_R w \sin \alpha/c)^2]^{1/2}}{1 + (v_R w \cos \alpha)/c^2} \, . \qquad (37)$$

If the particle moves parallel to the x and x' axes, then

$$v = (v_R + w) / (1 + v_R w/c^2) \, . \qquad (38)$$

It follows from this equation that the composition of two velocities that are smaller than c always results in a velocity that is smaller than c....It also follows that the velocity of light c cannot be changed by compounding it with a 'subluminal velocity.' For this case we get

$$v = (v_R + c) / (1 + v_R/c) = c \, .$$

The velocity addition theorem illustrates beautifully the Correspondence Principle. In this case the velocity addition of Newtonian relativity, $v = v_R + w$, is now understood as a special case in the more comprehensive paradigm.

...We have now derived the required laws of the kinematics corresponding to our two principles, and proceed to their application in electrodynamics.

We will follow Einstein there in Section B of his paper, the focus of this column's next installment.

References

[1] Albert Einstein, "Zur Elektrodynamik Bewegter Körper," *Annalen der Physik* **17**, 891-921 (1905).

[2] Einstein quotes in this annotation are from the translation by John Stachel (Ed. and trans.) and Roger Penrose, *Einstein's Miraculous Year: Five Papers that Changed the Face of Physics* (Princeton Univ. Press, 1998). This excellent translation includes a chapter of background and history for each of the five 1905 papers.

[3] An earlier translation of Einstein's 1905 relativity papers is W. Perrett and G. B. Jefferey (trans.), *The Principle of Relativity* (Methuen, 1923; reprinted by Dover, 1952). In 1912 Einstein wrote a summary of Special Relativity in which some of his 1905 calculations reappear. This was recently published with translation as *Einstein's 1912 Manuscript on the Special Theory of Relativity* (Braziller, 2003).

[4] The philosophy and physics of space, time, and spacetime has a long and rich history. A superb anthology of principal thinkers "from Augustine to Einstein" can be found in J. Smart (Ed.), *Problems of Space and Time,* McMillan (1964).

[5] E. Taylor and J. A. Wheeler, *Spacetime Physics* (Freeman, 1966 and 1992).

[6] Translation dictionary between this annotation and Einstein's paper of Ref. 1:

This annotation	**Einstein**
Lab frame	Rest system K
Rocket frame	Moving system k
c	V
v_R	v
(x, y, z, t)	(x, y, z, t)
(x', y', z', t')	(ξ, η, ζ, τ)

[7] See H. A. Lorentz's papers in the Methuen/Dover reprint of Ref. 3.

[8] Hay's experiment as described J. Bronowski, *The Ascent of Man*, Little & Brown (1973), p. 255. In the BBC television broadcast version of this scene, Bronowski stands over a record played on an antique gramophone as he describes Hay's experiment.

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